that they have spawned: Hausdorff Paradox, Hausdorff measure, Hausdorff dimension, Hausdorff matrices, Hausdorff summation method, Hausdorff–Toeplitz Theorem, Hausdorff–Young Inequality, Hausdorff moment problem. As for the selections from his Nachlaß, the editors have chosen to highlight his broad interests in analysis and his penchant for surprising counterexamples. One item reveals that Hausdorff discovered the "long line" in 1915, well before its appearance in a paper by P. Alexandroff in 1924. The ultimate selection has no deep mathematical significance, but it is of great poignancy. The calculation of a particular improper integral was produced by Hausdorff at the behest of his son-in-law Arthur König. It is dated January 16, 1942, and it is his last mathematical work. On January 26, 1942, Felix Hausdorff, his wife, and his wife's sister committed suicide when faced with internment at Endenich. Internment was a prelude to deportation to the East and almost certain death.

Parts II and III of Volume IV are devoted to the areas of algebra and number theory, respectively. Three articles appear in the algebra section: the first is a contribution to the study of finite-dimensional associative algebras; the second concerns the exponential formula for Lie algebras, and it establishes the Baker–Campbell–Hausdorff formula; the third concerns what are now called Clifford algebras. A Nachlaß entry on finite commutative rings completes Part II. There is only one article classified as number theory, a simplification of Hilbert's solution of Waring's Problem. Hausdorff's review of Landau's *Handbuch der Lehre von der Verteilung der Primzahlen* [Landau, 1909] is also reprinted in Part III.

The Hausdorff Edition is a very ambitious project. The challenges faced by the editors are clearly illustrated in these two volumes. Felix Hausdorff was a skilled writer. His authorial prowess is quite evident in *Grundzüge der Mengenlehre*. In Volume II, the editors let the text speak for itself. However, their annotations help provide context for today's reader. Walter Purkert's historical introduction and the following commentaries by the editors on set-theoretical and topological concepts help frame *Grundzüge*'s contributions and make manifest the extent of its influence on subsequent generations. In Volume IV, the many separate articles with their focus on specific problems (this being the nature of most research publications) threaten to obscure the overall contribution of this body of work. In this case, the editors have successfully diffused the threat by providing detailed commentaries, usually immediately following a given article; the commentaries do not shy away from clear-eyed judgments of a work's importance and historical significance. Finally, the editors' selections from Felix Hausdorff's Nachlaß help to show what a rich resource this collection promises to be for further scholarly research. These books and all the succeeding volumes of Hausdorff's collected works should be on the shelves of our research libraries.

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Available online 12 June 2006

10.1016/j.hm.2006.04.005

Cogwheels of the Mind. The Story of Venn Diagrams
By A.W.F. Edwards. Baltimore and London (The Johns Hopkins University Press). 2004. ISBN 0-8018-7434-3.
xvi + 110 pp. $25

Anthony Edwards has written an engaging, very readable, and profusely illustrated account of the evolution of Venn diagrams from their inception in 1880 to the present day. Containing no fewer than 50 images (nearly all in color, including a full-length portrait of John Venn), his study covers the period up to 2003, when the solution of an important open problem, a proof of the existence of nonsingular symmetrical n-set diagrams for all primes n, was reported. Included in this nearly 125-year history are accounts of important work done by Kiran B. Chilakamarri, Branko Grünbaum, Peter Hamburger, Raymond E. Pippert, D.W. Henderson, and Edwards himself (see, for example, Grünbaum, 1975).

In the preface, the author explains that he wrote the book to provide both a popular and accurate account of Venn diagrams from a *geometrical* rather than a logical viewpoint. In Chapter 3 we are told the meaning of the book's title: "Cogwheels of the Mind" refers to the author's method of extending Venn diagrams by adding each new set boundary relative to the same original set boundary, in contrast to the method of Venn and his successors, who added each new set boundary relative to the last one added. The former method results in Edwards–Venn diagrams. Visually, the latter method compounds the number of teeth in the resulting "comb-like" shape, whereas the Edwards–Venn method adds only the number of teeth or "cogs" that the additional set needs.

In Chapter 3 Edwards discusses the various attempts to construct a Venn diagram for five or more sets while adhering to Venn's original idea for his diagrams: "All that we have to do is to draw our figures, say circles, so that each successive one which we introduce shall intersect once, and once only, all the subdivisions already existing, and we then have what may be called a general framework indicating every possible combination producible by the given class terms" [Venn, 1880, 5]. But beyond four sets, Venn was unable to preserve this idea in his construction.

Discussing the diagrams of one of Venn's contemporary logicians, Lewis Carroll, in his book *Symbolic Logic, Part I* (1896) (see Bartley, 1986), Edwards claims, "[W]hen Carroll comes to a fifth set his courage fails him..." (p. 20). "[I]t is an admission of failure that he, like Venn, has not solved the problem in terms of true Venn diagrams" (p. 34). In fairness, Carroll's purposes were somewhat different from Venn's and very different from Edwards'. As Edwards explains in Chapter 1 of his book, Venn set out to improve on Leonhard Euler's diagrams and to represent George Boole's method, i.e., the complicated propositions resulting from his theory of development, which involve classes that are only exclusionary. Carroll, in contrast, had three goals: First, he wanted to develop a construction that could *easily* be generalized to more than the six sets Venn had described. Second, he wanted his diagrams to have the ability to identify whether or not a cell was empty or occupied, in order to represent not only universal propositions, but *existential* ones as well, a feature that Venn could not include in his diagrams. Finally, like Venn, he wanted to use his diagrams as a technique for solving complicated syllogistic arguments where it becomes important to easily identify and erase cells representing classes that are destroyed in working through the premises of an argument.

Threading through the first three chapters is a description of the important role played by the 19th-century Oxford mathematician Henry J.S. Smith, when he suggested the possibility of drawing Venn diagrams for an arbitrary number of sets on the surface of a sphere. Edwards provides many illustrations of the ubiquitous presence of Venn's diagram for three sets, especially in Chapter 2, and there lays the groundwork for the development of the rest of the book.

The really interesting part begins with Chapter 4, where Edwards discusses his own Edwards–Venn diagrams in connection with the correspondence between a Venn diagram and two combinatorial algorithms: the Gray code (to list all the ways of selecting among n different things in an order which guarantees that successive selections differ in just one of the things) and the "revolving-door algorithm" (to list all the ways of choosing r things from n different things).

In Chapter 5, Edwards connects the insight he gained from two observations made by the London biometrician C.A.B. Smith, the first concerning a Mercator-style diagram of the sets representable by a certain family of cosine curves that maps a Gray code, and the second, the corresponding family of sine curves that generates a Venn diagram that maps the binary numbers in their natural order. These observations lead to a discussion of *simple* Venn diagrams.

In the penultimate Chapter 6, Edwards discusses his own work from 1990, demonstrating that the dual of a Venn diagram is a maximal planar subgraph of a Boolean cube. And in the final chapter, we are treated to a blizzard of results that extend the theory of Venn diagrams to include new forms: those that are irreducible, symmetrical, and completely symmetrical. It is in this chapter that Edwards' own work shines.

Some surprising correctives are described. It seems that in 1971 Martin Gardner had published a Venn diagram for an arbitrary number of sets in his *Scientific American* column [Gardner, 1971], a fact that went unnoticed. And in his 1880 paper Venn claimed that ellipses cannot be used as geometrical figures to extend a four-set diagram to a five-set one that would preserve the essential feature of having no cells disjoint [Venn, 1880, 7]. In 2000, Hamburger and Pippert showed the claim to be false by constructing a simple reducible five-set diagram using five congruent ellipses [Hamburger and Pippert, 2000].

Syntactical and semantical errors are few in number. In the first category, on p. 69, the word "form" has been incorrectly inverted to "from"; on p. 30, the redrawing of Venn's five-set diagram is incorrect; and there is an incorrect sentence in the middle of p. 43 that should be removed. (Edwards identified the latter two errors in a private message, adding that they will be corrected in the reprint of the book.) The index is adequate, but not more than that. In the second category, Edwards' algorithm for constructing Carroll diagrams, although combinatorially correct, is too general to generate them (p. 33). In a private message, Edwards commented, "Every statement [on p. 33] seems to be true. All it lacks is an explanation of why Carroll chose the particular partitions he did…"

There are two appendices, each containing novel material. The first concerns a statistical application (contingency tables) of Carroll's three-set diagram that utilizes the *areas* of cells, from an article published in 1992 by the author and J.H. Edwards, and reprinted here. The second deals with the author's construction of a rotatable Edwards–Venn diagram that he presented in 1989.

Edwards first published his diagrams in 1989 in an article appearing in the *New Scientist* [Edwards, 1989]. Many new developments have occurred since that publication and they are addressed in this slim book. Writing in a style that makes the book accessible to a wide range of readers, his approach to Venn diagrams will appeal to those with a historical bent interested in combinatorial geometry where aesthetics and open problems are important, as well as to logicians, whose field originally inspired Venn to construct his diagrams.

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Available online 19 December 2005

10.1016/j.hm.2005.10.001